VUB WETENSCHAPPEN & BIO-INGENIEURSWETENSCHAPPEN

The Research Group of

ALGEBRA (ALGB)

has the honour to invite you to the public defence of the PhD thesis of

Maya VAN CAMPENHOUT

to obtain the degree of Doctor of Sciences (Mathematics)

Title of the PhD thesis:

Finitely generated algebras defined by homogeneous quadratic monomial relations and their underlying monoids

Promotor:

Prof. Dr. Eric Jespers

Prof. Dr. Stefaan Caenepeel

The defence will take place on

September 30, 2016 at 16:00

in Auditorium D.2.01 at the Campus Etterbeek of the Vrije Universiteit Brussel, Pleinlaan 2 in 1050 Elsene, and will be followed by a reception

Members of the jury

Prof. Dr. M. SIOEN (VUB, chairman)
Prof. Dr. K. DE COMMER (VUB, secretary)
Prof. Dr. E. JESPERS (VUB, promotor)
Prof. Dr. S. CAENEPEEL (VUB, promotor)
Prof. Dr. P. CARA (VUB)
Prof. Dr. A. DOOMS (VUB)
Prof. Dr. F. CEDÓ (UAB, Spain)
Prof. Dr. J. OKNIŃSKI (Univ. Warsaw, Poland)
Dr. A. BÄCHLE (VUB)

Curriculum vitae

Maya Van Campenhout (°September 4, 1987) graduated as a master in Mathematics in July 2010, at the Vrije Universiteit Brussel. Subsequently, she started her PhD in the research group ALGB (Vrije Universiteit Brussel), under the supervision of Prof. Dr. E. Jespers and Prof. Dr. S. Caenepeel. She attended three international scientific conferences and gave several oral presentations. She is the (co-)author of one peer-reviewed paper published in Journal of Algebra, and of one submitted paper (Journal of Algebra).

Abstract of the PhD Research

We consider algebras over a field with finitely many generators x_1, $x_2,\dots, x_n$ subject to n choose 2 monomial relations of degree two. More precisely, such an algebra is the semigroup algebra K[S] of a monoid, called a quadratic monoid, $S=\langle x_1, x_2,\dots,x_n \mid R\rangle$, where R is a finite set of relations that are of the type $x_ix_j= x_kx_l$ with $(i,j)\neq(k,l)$ and moreover, every word x_ix_j appears at most once in all the relations. If these relations are non-degenerate then it is shown that the algebra K[S] is left and right Noetherian, has Gelfand-Kirillov dimension at most n and satisfies a polynomial identity. In case the defining relations are square free this was already established by Gateva-Ivanova, Jespers and Okniński. To prove these results we investigate the structure of the underlying monoid S.

Moreover, we show that there is a strong link with the divisibility monoids and monoids of I-type. Monoids of I-type are examples of cancellative non-degenerate quadratic monoids. It is known that their semigroup algebras are Noetherian domains. They are a monoid interpretation of non-degenerate involutive set-theoretic solutions of the Yang-Baxter equation and have been studied quite intensively in recent years, by Gateva-Ivanova and Van den Bergh, Etingof, Schedler and Soloviev, Jespers and Okniński. Divisibility monoids have been introduced by Kuske. These are cancellative monoids that include the class of monoids of I-type. We show that if they have a presentation with the maximal number of defining relations, then they are monoids of I-type.

Furthermore, we show that a non-degenerate quadratic monoid S contains an abelian submonoid A that is finitely generated and that K[S] is a finite module over the Noetherian commutative algebra K[A]. We show that S is of I-type if and only if S is cancellative and satisfies the cyclic condition. Furthermore, if S satisfies the cyclic condition, then S is cancellative if and only of K[S] is a prime ring. Moreover, in this case, one can replace the monoid A by a finitely generated submonoid A', whose normalizer equals S. In particular K[S] is a normalizing extension of K[A'] and thus the prime ideals of K[S] are determined by the prime ideals of K[A'].

These investigations are a continuation and generalization of earlier results of Cedó, Gateva-Ivanova, Jespers and Okniński in the case the defining relations are square free.