SCIENCES & BIOENGINEERING SCIENCES

The Research Group

Algebra, Incidence Geometry (ALGB)

has the honour to invite you to the public defence of the PhD thesis of

Arne Van Antwerpen

to obtain the degree of Doctor of Science

Title of the PhD thesis:

The Algebra of the Yang-Baxter equation

Promotors:

Prof. dr. Eric Jespers

Dr. Andreas Bächle

The defense will take place on

Friday, July 10, 2020 at 16h

for a limited audience. The defense will be livestreamed. Contact Kenny.de.commer@vub.be for more information.

Members of the jury

Prof. Dr. Stefaan CAENEPEEL (VUB, chairman)
Prof. Dr. Kenny DE COMMER (VUB, secretary)
Prof. Dr. Ann DOOMS (VUB)
Prof. Dr. Ir. Luc DE VUYST (VUB)
Prof. Dr. Adolfo BALLESTER-BOLINCHES (Universitat de València, Spain)
Prof. Dr. Victoria LEBED (Université de Caen, France)
Prof. Dr. Eric JESPERS (VUB, promotor)
Dr. Andreas BÄCHLE (VUB, promotor)

Curriculum vitae

Arne Van Antwerpen was born on the 7th of July 1993 in Edegem. He obtained his BSc. in Mathematics at the Universiteit Antwerpen in 2014 and his MSc. in Mathematics in 2016 at the same university. In October 2016 he started his doctoral investigations at the research group Algebra and Incidence geometry (ALGB) at Vrije Universiteit Brussel, under the supervision of Prof. Dr. Eric Jespers and Dr. Andreas Bächle, as Aspirant fundamenteel onderzoek of FWO-Flanders. Arne Van Antwerpen co-authored six articles, published in peer-reviewed international journals. He is sole author of one of these. He organized several research visits in Shanghai, Lecce and Buenos Aires. During his doctoral investigations he spoke at 10 international congresses and seminars.

Abstract of the PhD research

The Yang-Baxter equation pops up as a shadow emperor in several areas of physics and mathematics. It originated in work of Yang and Baxter on statistical physics and appears as a fundament of knot theory, where it corresponds to the third Reidemeister move, *i.e.* one of the three actions you can perform on a knot without changing it fundamentally. Due to its fundamental nature, the search of solutions of the Yang-Baxter equation has been extensive and created the notion of quantum groups. In this context Drinfel'd proposed to search for set-theoretic solutions of the Yang-Baxter equation, *i.e.* sets $\boldsymbol{X}$ and maps $\boldsymbol{r} : \boldsymbol{X} \times \boldsymbol{X} \to \boldsymbol{X} \times \boldsymbol{X}$ such that on $\boldsymbol{X}^3$ it holds that

$$(\boldsymbol{id}_X \times \boldsymbol{r})(\boldsymbol{r} \times \boldsymbol{id}_X)(\boldsymbol{id}_X \times \boldsymbol{r}) = (\boldsymbol{r} \times \boldsymbol{id}_X)(\boldsymbol{id}_X \times \boldsymbol{r})(\boldsymbol{r} \times \boldsymbol{id}_X).$$

The study of important classes of such solutions can be reduced to the study of skew left braces, introduced by Rump, Guarnieri and Vendramin, which are sets with two algebraic structures. These are operations, satisfying a skew version of the left distributive law. In this thesis we study the structure and properties of skew left braces with inspiration stemming from group theory, the mathematical study of symmetry, and ring theory, the study of sets with two operations that behave like the addition and multiplication of numbers. In particular, we examine what properties skew left braces have, if we know they are built from two well-understood skew left braces. Furthermore, we introduce a radical of skew left braces, i.e. a set of "bad elements", which we relate to the so-called weight of a skew left brace.

Another approach to study set-theoretic solutions is to examine their so-called structure group and structure monoid. These are algebraic structures that encode the behavior of the set-theoretic solution $(\boldsymbol{X}, \boldsymbol{r})$. To be precise, they are the group $\boldsymbol{G}(\boldsymbol{X}, \boldsymbol{r})$ (resp. the monoid $\boldsymbol{M}(\boldsymbol{X}, \boldsymbol{r})$) defined by the following presentation

$$\langle \boldsymbol{x} \in \boldsymbol{X} \mid \boldsymbol{xy} = \boldsymbol{uv} \textbf{ if } \boldsymbol{r}(\boldsymbol{x}, \boldsymbol{y}) = (\boldsymbol{u}, \boldsymbol{v}) \rangle.$$

We show that the monoid behaves particularly nice. It is Noetherian and Abelian-by-finite, which means that it does not grow fast and it behaves in spirit like the set of positive integers. A last feat of arms of this approach is the study of the behavior of the algebra $\boldsymbol{KM}(\boldsymbol{X}, \boldsymbol{r})$ over any field and we determine its prime ideals.

In the last part of the thesis we focus on units of group rings and contribute to the study of Coleman automorphisms of finite groups.