SCIENCES & BIOENGINEERING SCIENCES

The Research Group

**Algebra & Analysis**

has the honor to invite you to the public defense of the PhD thesis of

# Charlotte Verwimp

to obtain the degree of Doctor of Sciences

Title of the PhD thesis:

**Set-theoretic solutions of the Yang-Baxter equation and associated algebraic structures**

Promotoren:
**Prof. dr. em. Eric Jespers (VUB)**
**Prof. dr. Leandro Vendramin (VUB)**

The defense will take place on
**Friday, May 20, 2022 at 16h00 in auditorium D.2.01**

The defense can also be followed through a live stream. Please contact Charlotte.Verwimp@vub.be for more information.

## Members of the jury

Prof. dr. Ann Dooms (VUB, chair)
Prof. dr. Kenny De Commer (VUB, secretary)
Dr. Arne Van Antwerpen (VUB)
Prof. dr. Dominique Maes (VUB)
Prof. dr. Tomasz Brzezinski (University of Swansea)
Prof. dr. Victoria Lebed (University of Caen)

## Curriculum vitae

Charlotte Verwimp obtained her Bachelors degree in Mathematics in 2016 and her Masters degree in Mathematics in 2018 at the Vrije Universiteit Brussel, graduating summa cum laude. Afterwards she started her doctoral studies at the research group Algebra & Analysis under the supervision of Prof. Dr. em. Eric Jespers and Prof. Dr. Leandro Vendramin, supported by an FWO Fellowship for fundamental research. Her research on set-theoretic solutions of the Yang-Baxter equation was published in several peer-reviewed international journals and has been presented internationally at conferences and workshops.

## Abstract of the PhD research


The Yang-Baxter equation is one of the essential equations in mathematical physics, initially appearing in both quantum and statistical mechanics. The problem of constructing and classifying its solutions has been fruitfully approached by Drinfeld who proposed the idea to focus on the subclass of set-theoretic solutions. So far, not all such solutions of the Yang-Baxter equation are known.

This PhD thesis is highly motivated by this open problem. An effective approach is to identify and study the underlying algebraic structures. More precisely, we focus on the (semi)group and ring theoretical aspects that occur, and study them for specific classes of set-theoretic solutions of the Yang-Baxter equation.

Initially, we deal with arbitrary set-theoretic solutions of the Yang-Baxter equation and discover connections, via 1-cocycles, between three (in general different) monoids associated to a set-theoretic solution, the structure monoid and the left and right derived structure monoids.

In case the set-theoretic solution is left non-degenerate, the 1-cocycle between the structure monoid and the left derived structure monoid is bijective. This allows us to put two monoid structures on a same set, leading to the definition of a YB-semitruss. YB-semitrusses turn out to be the suitable associative algebraic structure to study left non-degenerate set-theoretic solutions of the Yang-Baxter equation. In particular, they can be used to prove that any such finite solution is right non-degenerate if and only if it is bijective, which is one of the main results in this thesis. If a solution is (left and right) non-degenerate and bijective, we determine when its structure monoid and derived structure monoids are Malcev nilpotent, and deal with multipermutation solutions.

Set-theoretic solutions that are not left nor right non-degenerate are explored to a much smaller extent. In the final part of this thesis, such solutions are generated using the theory of skew lattices. Moreover, the obtained set-theoretic solutions turn out to be idempotent or cubic.