VUB SCIENCES & BIOENGINEERING SCIENCES

The Research Group

Algebra, Incidence Geometry (ALGB)

has the honor to invite you to the public defense of the PhD thesis of

Geoffrey JANSSENS

to obtain the degree of Doctor of Sciences

Title of the PhD thesis:

Identities of Affine Algebras and Their Asymptotic Behaviour

Promotor:

Prof. dr. Eric Jespers

The defence will take place on

Friday 19 May 2017 at 16:00 h

in Auditorium D.2.01 at the Campus Etterbeek of the Vrije Universiteit Brussel, Pleinlaan 2 - 1050 Elsene, and will be followed by a reception.

Members of the jury:

Prof. Dr. Stefaan Caenepeel (chairman)
Prof. Dr. Kenny De Commer (secretary)
Dr. Alexey Gordienko (co-promotor)
Prof. Dr. Ann Dooms
Prof. Dr. Dominique Maes
Dr. Andreas Bächle
Dr. Spela Spenko
Prof. Dr. Eli Aljadeff
(Israel Institute of Technology)
Prof. Dr. Mikhail Zaicev (Moscow State Univ.)

Curriculum vitae

In 2012, Geoffrey Janssens obtained a Master degree in Fundamental mathematics at the VUB. After two years as assistent at the mathematics departement, he started a Ph.D, as an aspirant from the Fund for Scientific Research Flanders (FWO), under the supervision of Prof. Eric Jespers.

The resulting research was published in several peer-reviewed journals and has been presented internationally at conferences and workshops.

Abstract of the PhD research

Cooking and tasting food, washing and drying clothes, going left or right while driving, stock exchange, momentum and position of subatomic particles in quantum mechanics; our everyday life and nature has an abundance of noncommutative operations. Many of these phenomena can be modelled by (non-commutative) algebraic structures called algebras.

One of the main goals in this thesis is to study these objects in some generic way. More precisely how can one detect that two of these algebras are models of a same instance? What do the 'rough shapes' of these abstract objects look like? In this thesis we investigate how the knowledge of so-called polynomial identities yields insight into the 'rough shapes' of algebras.

For example: the real numbers commute, i.e. $ab = ba$ for any pair of real numbers a and b. In other words: all real numbers are related by the relation $xy - yx = 0$, so if we substitute any real numbers a and b in the equation $xy - yx = 0$ the output will always be a true statement. On the other hand, momentum M and position P do not commute since $PM - MP \neq 0$. Also matrices, for example, do not commute. Thus, for a set of objects/functions to commute is a strong condition that gives a first very rough glance on the form of the algebra that models the set of objects under consideration.

Commutativity is a relation/condition that has the form of a polynomial in non-commuting variables (i.e. we may not simply interchange the variables). If this relation is satisfied, we call the polynomial a polynomial identity of the algebra.

These polynomial identities have the advantage of yielding connections with other fields of mathematics such as combinatorics, representation theory and geometry. In this thesis we combine these fields to associate invariants to any PI algebra and use them to, possibly, distinguish different algebras. Among other, we get an interpretation of the polynomial growth rate, introduce codimensions for rings, include semigroup gradings and show which obstructions appear.