

## ALGB: Algebra, Incidence Geometry

has the honor to invite you to the public defense of the PhD thesis of

**Doryan TEMMERMAN**

to obtain the degree of Doctor of Sciences

Title of the PhD thesis:

Fixed point properties for low rank linear groups over orders  
and applications to integral group rings.

### Promotors:

Prof. dr. Eric Jespers  
Dr. Geoffrey Janssens

The defence will take place on

**Friday May 24 2019 at 16:00h**

in Auditorium E.0.12 at the Campus  
Humanities, Sciences and Engineering of the  
Vrije Universiteit Brussel, Pleinlaan 2 - 1050  
Elsene, and will be followed by a reception.

### Members of the jury:

Prof. dr. Stefaan Caenepeel (chairperson)  
Prof. dr. Ann Dooms (secretary)  
Prof. dr. Ben Craps  
Dr. Sugandha Maheshwary (IISERM)  
Prof. dr. Alain Valette (Université de  
Neuchâtel)  
Prof. dr. Ángel del Río (Universidad de Murcia)

### Curriculum vitae

Doryan Temmerman (°February 19, 1992, Vilvoorde, Belgium) obtained his Master in (Fundamental) Mathematics at the VUB in 2015, graduating summa cum laude. Afterwards he started his doctoral studies at the ALGB research group via an FWO Fellowship for fundamental research. His research focused on free constructions in the group of units of integral group rings and geometric properties of the latter, which has so far led to a publication in the Proceedings of the American Mathematical Society and multiple preprints (to appear in international journals). He presented his work at multiple international conferences and was a long-term visitor at the USP in São Paulo.

### Abstract of the PhD research

In the study of symmetries in mathematics, groups, i.e. sets with a binary operation satisfying some specific rules, play an important role. Many questions pop up when one tries to represent these groups as something a computer may calculate with: matrices over rings. This is done in the so-called representation theory of groups. One such question is whether these reductions to matrices allows us to reconstruct the group. In other words, can we have two different groups which have exactly the same representations as matrices over some ring? In general, the answer, counterintuitively, appears to be yes. However, the study of what classes of groups one can distinguish is still very much open. One way to tackle this problem is to study the unit group  $U(ZG)$  of an integral group ring  $ZG$  over a group  $G$  and determine how rigidly  $G$  lies in this structure.

In the first part of this thesis, we provide a way of finding many “copies” of a subgroup  $H$  of  $G$  inside of  $U(ZG)$ , via a relatively new construction called the Bovdi units. We study the structural properties that several of such copies together can possess. We prove that when  $H$  is cyclic we often can construct two of these copies such that there is almost no interplay between them.

A second main part of the thesis is trying to understand this rigidity when considering  $U(ZG)$  as the symmetries of a tree. In this mathematical context, a tree is a graph without loops. It appears that some structural information can be gained on  $U(ZG)$  if it has symmetries on a tree such that no point is fixed by all the symmetries at once. The converse of this property is called (FA), after “points Fixes sur les Arbres”. We prove a result which reduces the study of a small adaptation of (FA) (called (HFA)) to smaller components of  $U(ZG)$ .

The study of (HFA) for these smaller groups, which are linear groups over orders, form the bulk of thesis. We focus on the lower rank case since these form the biggest gap in the state-of-the-art. We deduce exactly when  $U(ZG)$  has (HFA) and find a list of 10 minimal counter examples.