

## Topological Algebra, Functional Analysis and Category Theory

has the honor to invite you to the public defense of the PhD thesis of

**Karen VAN OPDENBOSCH**

to obtain the degree of Doctor of Sciences

Joint PhD with Universiteit Antwerpen

Title of the PhD thesis:

A study of topological properties in approach theory using monoidal topology

### Promotors:

Prof. Eva Colebunders  
Prof. Mark Sioen  
Prof. Bob Lowen

The defense will take place on

**Friday February 1 2019 at 15:00h**

in Auditorium D.2.01 at the campus Humanities, Sciences and Engineering of the Vrije Universiteit Brussel, Pleinlaan 2 - 1050 Elsene, and will be followed by a reception.

### Members of the jury:

Prof. Stefaan Caenepeel (chairman)  
Prof. Alain Verschoren (secretary)  
Prof. Maria Manuel Clementino (univ. de Coimbra)  
Prof. Frédéric Mynard (New Jersey City Univ.)

### Curriculum vitae

Karen Van Opdenbosch (°17/01/1990) obtained a Master of Science in Mathematics from the VUB in 2013. After graduation, she became a teaching assistant at the department of Mathematics of the VUB and started a joint PhD between the research group TACT of the VUB and the research group Fundamental Mathematics of the UA. Her research focused on approach theory from the point of view of monoidal topology. Her research so far led to the publication of 3 articles published in peer-reviewed journals and the research has been presented nationally and internationally at conferences and workshops.

### Abstract of the PhD research

Mathematical models involving metric spaces are used in many branches of science. Metric spaces  $(X, d)$  with  $X$  a set and  $d$  a metric, provide the possibility of expressing convergence of a sequence  $(x_n)_n \rightarrow x$  in  $X$ , while at the same time having the power to express numerical data such as calculating how far a point  $y$  is from being a convergence point of  $(x_n)_n$ . The most important example is the real line  $\mathbb{R}$  endowed with the Euclidean metric  $d_E$ .

When also realvalued functions  $f: Z \rightarrow \mathbb{R}$  have to be modeled, with convergence of sequences  $(f_n)_n \rightarrow f$  in  $X = \mathbb{R}^Z$ , like for instance pointwise convergence, function spaces are needed. One can express pointwise convergence by using an appropriate topology on  $\mathbb{R}^Z$ , but there is no canonical metric describing this convergence. While the topology describes the right convergence notion, all numerical data is lost, so one drops from the numerical setup of  $(\mathbb{R}, d_E)$  to a non-numerical topological setup in  $\mathbb{R}^Z$ .

Approach theory completely solves this. Instead of axiomatizing the distance  $d(x, y)$  between points of  $X$  (like in a metric space), approach theory provides axioms for a distance  $\delta(x, A)$  between points  $x$  and subsets  $A$  of  $X$ . Starting with the Euclidean metric  $d_E$  on  $\mathbb{R}$  and its associated distance  $\delta_E(x, A) = \inf_{\{a \in A\}} d_E(x, a)$ , a canonical distance  $\delta(f, \vartheta)$  from a function  $f$  to a subset  $\vartheta$  of  $\mathbb{R}^Z$  does exist, having the capacity to describe pointwise convergence  $(f_n)_n \rightarrow f$  in  $X = \mathbb{R}^Z$  and having the power to express numerical data, such as calculating how far a function  $g$  is from being a convergence point of  $(f_n)_n$ .

Similar principles have proved useful in such diverse areas as functional analysis, probability theory and theoretical computer science, meriting a deeper study of the theoretical foundations of the theory.

As such the thesis studies possible links between approach theory and monoidal topology, a research area providing a unifying framework on how to axiomatize “spaces” in terms of convergence. We look for appropriate monads and quantales to describe the category of approach spaces and its subcategory of non-Archimedean spaces and investigate convergence of functional ideals, which is the key to our description of approach spaces as relational algebras for the functional ideal monad. This description is the main instrument for an in depth study of new approach invariants.