UNCERTAINTY PRINCIPLES

Loosely speaking, uncertainty principles in mathematics may be considered as statements expressing that *a function and its Fourier transform cannot simultaneously be very small at infinity unless the function is identically zero.* A simple and clear manifestation of this principle is that a non-zero function and its Fourier transform cannot both have compact support. A much deeper result of Hardy [2] from 1933 states that if a function and its Fourier transform are both $o(e^{-\frac{1}{2}x^2})$, then the function is identically zero. As of today, a wide variety of uncertainty principles has been established by using various different techniques (complex analysis, Hilbert space theory, ...). Furthermore, uncertainty principles often may be interpreted and imply physical limitations in other branches of science such as quantum mechanics and signal analysis.

The goal of this project is to study various uncertainty principles (according to the taste of the student) and learn more about the mathematics needed to prove them. Possible starting points are the books [1, 3, 4].

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References

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[3] V. Havin, B. Jöricke, *The uncertainty principle in harmonic analysis*, Springer Science & Business Media, 2012.

[4] P. Koosis, *The logarithmic integral I*, Cambridge University Press, 1988.