Abstract

Group representations: idempotents in group algebras and applications to units

In this thesis, we study the group $\mathcal{U}(RG)$ of units of RG, where R is the ring of integers of a number field F. In particular, we consider the group $\mathcal{U}(\mathbb{Z}G)$.
First, we investigate the primitive central idempotents and the Wedderburn decomposition of group algebras FG, with F a number field and G a strongly monomial group. Next, we focus on a complete set of matrix units in the Wedderburn components of $\mathbb{Q}G$ and $\mathbb{F}G$, with $\mathbb{F}$ a finite field, for a class of finite strongly monomial groups G containing some metacyclic groups.
We will also classify the finite groups G for which, given a fixed abelian number field F, the Wedderburn components of FG are not exceptional.
Thereafter, we study the central units $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$ for finite groups G. We construct generalized Bass units and show that they generate a subgroup of finite index in $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$, for finite strongly monomial groups G. For a specific class within the finite abelian-by-supersolvable groups G, we can even describe a multiplicatively independent set (based on Bass units) which generates a subgroup of finite index in $\mathcal{Z}(\mathcal{U}(\mathbb{Z}G))$. For a different class of finite strongly monomial groups, containing some metacyclic groups, we construct such a set of multiplicatively independent elements starting from generalized Bass units.
Finally, we combine all results to construct a generating set of $\mathcal{U}(\mathbb{Z}G)$ up to finite index, for some classes of finite groups G.