WHITNEY'S EXTENSION THEOREM

A simple but deep problem in mathematical analysis is to characterise the functions defined on a closed set in $\mathbb{R}^d$ that may be extended as an m-times differentiable function, $m \in \mathbb{N}$, throughout the whole $\mathbb{R}^d$. One of the main difficulties is that closed sets are often too thin to uniquely determine the derivatives of functions on them (consider e.g. singletons or hyperplanes). In his influential work [2] from 1934, Whitney overcame this problem by considering not only a function but a function together with all its derivates (of order $\leq m$) as given data. More precisely, he characterised the tuples $(f^{(\alpha)})_{|\alpha|\leq m} \in \prod_{|\alpha|\leq m} C(X)$, $X \subseteq \mathbb{R}^d$ closed, that are the restriction of the partial derivatives of an m-times differentiable function defined on the whole $\mathbb{R}^d$ in terms of the formal Taylor polynomials of $(f^{(\alpha)})_{|\alpha|\leq m}$

$$T_x^m f(y) = \sum_{|\alpha|\leq m} \frac{f^{(\alpha)}(x)}{\alpha!}(y-x)^\alpha, \qquad x \in X, y \in \mathbb{R}^d.$$

This result is known as *Whitney's extension theorem* and may be considered as an inverse to Taylor's theorem.

The goal of this project is to study and discuss Whitney's extension theorem, mainly by following the exposition of Malgrange [1]. If time permits, further related topics could also be studied (construction of explicit extension operators on special types of closed sets, equivalence of sup-norms and Whitney-norms, ...).

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References

[1] B. Malgrange, *Ideals of differentiable functions*, Oxford University Press, 1967.

[2] H. Whitney, *Analytic extensions of differentiable functions defined in closed sets*, Trans. Amer. Math. Soc. **36** (1934), 63–89.