SCIENCES & BIOENGINEERING SCIENCES

The Research Group

ALGB: Algebra, Incidence Geometry

has the honor to invite you to the public defense of the PhD thesis of

Timmy FIEREMANS

to obtain the degree of Doctor of Sciences

Title of the PhD thesis:

Hopf and Frobenius $\mathcal{V}$-categories: Hopf-Galois Theory and the Larson-Sweedler Theorem in Enriched Category Theory

Promotor:

Prof. dr. Stefaan Caenepeel
Prof. dr. Joost Vercruysse (ULB)

The defense will take place on

Thursday September 26 2019 at 16:00h

in Auditorium D.2.01 at the Campus Humanities, Sciences and Engineering of the Vrije Universiteit Brussel, Pleinlaan 2 - 1050 Elsene, and will be followed by a reception.

Members of the jury:

Prof. dr. Mark Sioen (chairperson)
Prof. dr. Kenny De Commer (secretary)
Dr. Ana Agore
Dr. Špela Špenko
Prof. dr. Gert Desmet
Prof. dr. Yinhuo Zhang (UHasselt)
Dr. Christina Vasilakopoulou (Univ. of California)

Curriculum vitae

Timmy Fieremans (°February 9, 1990, Jette, Belgium) obtained his Master in (Fundamental) Mathematics at the VUB in 2013, graduating magna cum laude. Afterwards he started his doctoral studies at the ALGB research group, as an assistant at the Department of Mathematics at the Faculty of Engineering. His research focused on generalizations of Hopf and Frobenius algebras and Hopf-Galois Theory, and has led so far to a publication in Journal of Algebra and its Applications and several preprints. He presented his work at multiple international conferences.

Abstract of the PhD research

Groups, i.e. sets with a binary operation satisfying some axioms, appear in numerous areas in and outside mathematics and should be understood as the algebraic counterpart of symmetries in geometry. In a similar way, the notion of a Hopf algebra appears naturally in several branches of mathematics, such as homological algebra, Lie groups, topology, functional analysis, quantum theory, Hopf-Galois theory… Roughly speaking one can think of a Hopf algebra as follows: a Hopf algebra relates to a vector space, as a group does to a set. I.e. a Hopf algebra is a vector space, endowed with some operations that satisfy suitable axioms. The similarity with groups is very strong and can even be made formal. In philosophical terms, Hopf algebras provide an algebraic description of "hidden symmetries" that one can't describe with usual groups.

Because of their importance, a growing number of variations on the notion of a Hopf algebra have surfaced. One of these notions, called Hopf $\mathcal{V}$-category, was introduced recently as a linearized version of groupoids. Roughly speaking groupoids consist of several groups that are 'glued together' into what one could refer to as a 'many-object' group.
In the same way,
Hopf $\mathcal{V}$-categories can be seen as a many-object generalization of Hopf algebras.

Another algebraic notion closely related to a Hopf algebra is that of a so called Frobenius algebra. Frobenius algebras have the useful property that their category of modules (or representations) and their category of comodules (or corepresentations) are isomorphic.
The classical Larson-Sweedler theorem characterizes finite dimensional Hopf algebras among bialgebras as those that possess a non-singular left integral. The existence of an integral implies in particular that the Hopf algebra is Frobenius. Just as the notion of a Hopf algebra has been generalized in several ways, so has the Larson-Sweedler theorem.

In the first part of this thesis we give an answer to what the Frobenius counterpart is for Hopf $\mathcal{V}$-categories. We call them Frobenius $\mathcal{V}$-categories and provide several equivalent characterizations in order to proof a Larson-Sweedler type theorem for Hopf $\mathcal{V}$-categories.
The second part of this thesis gives a generalization of the Descent and Galois theory for k-linear Hopf-categories.
The question of descent theory is the following: Given $i: A \to B$ be a ring extension, for which right A-module M does there exist a right B-module N such that $M = N \otimes_B A$? This problem has an elegant solution in case of classical Galois extensions: there exists a unique module N with the desired property if and only if M allows a compatible action of the Galois group.
More generally, one can consider Hopf-Galois extensions, where the action of the Galois group is replaced by a Hopf algebra (co)action. In this work, we generalize these descent and Galois theories for extensions of k-linear categories and coactions of k-linear Hopf categories.