VUB SCIENCES & BIOENGINEERING SCIENCES

The Research Group

Algebra & Analysis

has the honor to invite you to the public defense of the PhD thesis of

Wouter Van Den Haute

to obtain the degree of Doctor of Sciences

Joint PhD with Universiteit Antwerpen

Title of the PhD thesis:

Frame theoretic methods in topology and analysis

Promotors:
Prof. dr. Wendy Lowen
Prof. dr. Mark Sioen

The defense will take place on

Wednesday, August 25, 2021 at 17h00

Universiteit Antwerpen, Campus Middelheim, M.G.010, Middelheimlaan 1, 2020 Antwerpen.
Please register by mail in advance

The defense can be followed through a live stream. Contact Wouter.Van.Den.Haute@vub.be for more information

Members of the jury

Prof. dr. David Eelbode (UAntwerpen, chair)
Prof. dr. Jan De Beule (VUB, secretary)
Prof. dr. Kenny De Commer (VUB)
Prof. dr. Boris Shoikhet (UAntwerpen)
Prof. dr. Aleš Pultr (Charles University, Czech Republic)
Prof. dr. Walter Tholen (York University, Canada)

Curriculum vitae

Wouter Van Den Haute obtained a Master of Science in Mathematics from the VUB in 2014. He started as a teaching assistant and PhD student at the Mathematics and Data Science department of the VUB in 2015 and was a teaching assistant at the University of Antwerp from 2016, leading to a joint PhD. His research focused on the use of frame theoretic methods in topology and analysis and led to 4 articles which were published in international journals.

Abstract of the PhD research

The study of frames is also known as pointfree topology. As the name suggests, this is a way of studying spaces without (mentioning) points. This idea is more natural than one might initially think: when drawing a point on paper, we do not draw an actual point, but rather a “spot”, which can be reduced in size if that would be required to serve our purposes. In topology, spots of varying sizes are modelled by open sets, which become the building blocks of so-called frames. In many cases, it is possible to reverse this process by defining points as being frame homomorphisms to 2={0,1}.

Approach spaces on the other hand can be used to combine the advantages of both topological and metric spaces. Topological spaces behave better with respect to metric spaces in the sense that we can take arbitrary products of topological spaces which still behave well. On the other hand, topological spaces are more of the all-or-nothing-type. Two points can be either the same, or not. A point can be inside a subspace, or not. Here, metric spaces have the advantage: we can say exactly how far two points are apart from each other. Approach spaces remedy this by defining the distances between a point and a set, not between points. One of the characterizations of approach spaces is the lower regular function frame L, consisting of contractions to P=[0,∞], which is in fact a frame.

In this thesis, we first remark that whereas point-set distances and lower regular function frames L have been extensively studied, their ‘interior’ counterparts ι and U were left mostly ignored. We begin by defining and studying ι and also consider upper regular function frame U. We then consider another property of approach spaces which was not extensively studied yet, namely normality. Normality is an important tool in order to prove existence of extensions of function, such as Urysohn maps, Tietze's extension and Katetov-Tong's insertion.

We then remark that, as for contractions, the set of semicontinuous maps to P also defines a frame. Moreover, a frame of opens can be described equivalently by the set of continuous maps to 2. Given that the points of a frame are defined by the frame homomorphisms to 2, we can replace both instances of 2 by another space/frame F. We investigate the transitions between spaces and F-frames and find a connection with free distributive lattices.